ABSTRACT

An analysis of the Poiseuille-Couette flow of two immiscible fluids between inclined parallel plates is investigated. One of the fluids is assumed to be electrically conducting while the other fluid and channel walls are assumed to be electrically insulating. The viscous and Ohmic dissipation terms are taken into account in the energy equation. The coupled nonlinear equations are solved both analytically valid for small values of the product of Prandtl number and Eckert number ($\epsilon$) and numerically valid for all $\epsilon$. Solutions for large $\epsilon$ reveal a marked change on the flow and rate of heat transfer. The effects of various parameters such as Hartmann number, Grashof number, angle of inclination, ratios of viscosities, widths and thermal conductivities are presented and discussed in detail.

Keywords: Poiseuille-couette flow, Magnetohydrodynamic convection, Immiscible fluids, Inclined channel, Hartman number.

1. INTRODUCTION

Convective heat transfer in channels has been an important research topic for the last few decades because of its applications in solar technology, safety aspects of gas cooled reactors and crystal growth in liquids, etc. Multi-layer convection is also found in many physical sciences such as geophysics, astrophysics, atmospheric physics and many other places (Andereck et al. [1]). Degen et al. [2] further introduced examples of mantle convection, modeling of stellar interiors and the growth of encapsulated crystals. Despite the various fields where multi-layer convection plays a role, much about it is still unknown. In the process of manufacturing materials in industrial problems for example in crystal growth using the horizontal Bridgman technique, Oreper and Szekely [3] involved an electrically conducting fluid subjected to a magnetic field. In that case the fluid experiences a Lorentz force and its effect is to reduce the velocities, exerting further the influence on the rates of heat and mass transfer. It is, therefore, important to study the detailed characteristics of transport phenomena in such a process so that a product of good quality can be developed with improved design for the manufacturing processes. There are some experimental and theoretical studies on hydromagnetic aspects of two-fluid flow available in literature. The interest in these types of flow problems is due to their abundance in technological applications such as magnetohydrodynamic (MHD) devices, thermonuclear power generators and nuclear engineering. Hartmann flow of a conducting fluid and a non-conducting fluid layer contained in a channel has been studied by Shail [4]. His results predicted that an increase of the order 30% can be achieved in the flow rate for suitable ratios of heights and viscosities of the two fluids. Lohrasbi and Sahai [5] studied two-phase MHD flow and heat transfer in a parallel plate channel with the fluid in one phase being a conductor. These studies are expected to be useful in understanding the effect of the presence of a slag layer on heat transfer characteristics of a coal-fired MHD generator. Umavathi et al. [6,7] and Malashetty et al. [8-10] studied MHD two-fluid flow models in horizontal and inclined channels.

The study of Couette flow in a channel of an electrically conducting viscous fluid under the action of a transversely applied magnetic field has immediate applications in many devices such as MHD power generators, MHD pumps, accelerators, aerodynamic heat-
ing, electrostatic precipitation, polymer technology, petroleum industry, purification of crude oil and fluid droplet sprays. The problem of Couette flow is also considered important in transpiration cooling. In this process several engines can be protected from the influence of hot gases. This process is used in turbojet and rocket engines, like combustion chamber walls, exhaust nozzles and gas turbine blades. Makar [11] found the analytical solutions for magnetohydrodynamic plane Couette flow of an elasto-viscous fluid using Laplace transformations. Jha [12] discussed the problem of natural convection in unsteady MHD Couette flow. Later on Ogulu and Motsa [13] analyzed numerically the radiative heat transfer to magnetohydrodynamic Couette flow with variable wall temperatures. The problem of unsteady Couette-Poiseuille flow with temperature dependent physical properties in the absence of a uniform magnetic field was studied by Attia [14]. Recently Sanjeev Kumar [15] gave a mathematical model for the MHD Couette flow through porous medium with heat transfer. Attia [16] also studied the influence of temperature dependent viscosity and thermal conductivity on the transient Couette flow with heat transfer. All these studies [11-16] are pertained to one fluid model. Thus keeping in view the wide range of applications in geophysics and MHD generators, the objective of this paper is to study the problem of MHD Poiseuille-Couette flow of two immiscible fluids in an inclined channel.

2. MATHEMATICAL MODEL

The physical configuration (Fig. 1) consists of two infinite inclined parallel plates extending in the z and x-directions, making an angle $\phi$ with the horizontal. The two plates are maintained at different constant temperatures $T_{w1}$ and $T_{w2}$. The region $0 \leq y \leq h_1$ is occupied by a viscous incompressible fluid of density $\rho_1$, viscosity $\mu_1$ and thermal conductivity $\psi_1$. The region $-h_2 \leq y \leq 0$ is occupied by a different (immiscible) viscous, incompressible electrically conducting fluid of density $\rho_2$, viscosity $\mu_2$, thermal conductivity $\psi_2$ and electrical conductivity $\sigma$. A uniform magnetic field $B_0$ is applied normal to the plates. It is assumed that the magnetic Reynolds number is sufficiently small so that the induced magnetic field can be neglected, and the induced electric field is assumed to be negligible. The flow in both the regions is assumed to be unidirectional, steady, laminar and fully developed. The Oberbeck-Boussinesq approximation is employed for density variation and the flow in both regions is assumed to be driven by the same constant pressure gradient ($-\partial p/\partial x$). Further the upper plate moves with a constant velocity $U$, while the lower one is kept stationary. Under these assumptions, the governing equations of motion and energy are (Romig [17])

Region 1

\[
\mu_1 \frac{d^2 u_1}{dy^2} + \rho_1 g \beta_1 (T_1 - T_{w1}) \sin(\phi) \frac{dp}{dx} = 0
\]  

(1)

\[
\psi_1 \frac{d^2 T_1}{dy^2} + \mu_1 \left( \frac{du_1}{dy} \right)^2 = 0
\]  

(2)

Region 2

\[
\mu_2 \frac{d^2 u_2}{dy^2} + \rho_2 g \beta_2 (T_2 - T_{w2}) \sin(\phi) - \sigma B_0^2 u_2 = \frac{dp}{dx}
\]  

(3)

\[
\psi_2 \frac{d^2 T_2}{dy^2} + \mu_2 \left( \frac{du_2}{dy} \right)^2 + \sigma B_0^2 u_2 = 0
\]  

(4)

where $u$ is the x-component of velocity and $T$ is the fluid temperature. The subscripts 1 and 2 denote the values for Region 1 and Region 2, respectively. It is assumed that the fluid and thermometric boundary conditions are unchanged under the addition of electromagnetic field. The boundary conditions on velocity are no-slip conditions, requiring that the velocity must be the same as that at the wall. The boundary conditions on temperature are isothermal conditions, with two boundaries at constant different temperatures. In addition the continuity of velocity, shear stress, temperature and heat flux at the interface is assumed.

Since the upper plate is moving with a constant velocity $U$ and pressure gradient is non-zero, the flow is termed as Poiseuille-Couette flow. The boundary and interface conditions on velocity and temperature are

\[
u_1(h_1) = U \quad T_1(h_1) = T_{w1}
\]

\[
u_2(-h_2) = 0 \quad T_2(-h_2) = T_{w2}
\]

\[
u_1(0) = u_2(0) \quad T_1(0) = T_2(0)
\]

\[
\mu_1 \frac{du_1}{dy} = \mu_2 \frac{du_2}{dy} \quad \psi_1 \frac{dT_1}{dy} = \psi_2 \frac{dT_2}{dy} \quad \text{at} \quad y = 0
\]  

(5)
where \( T_{w1} > T_{w2} \) is assumed to avoid thermal instability. Eqs. (1) ~ (4) along with boundary and interface conditions (5) are made dimensionless by using the following transformations

\[
\begin{align*}
\hat{u}_1 &= \frac{u_1}{U}, & \hat{u}_2 &= \frac{u_2}{U}, & \hat{y}_1 &= \frac{y_1}{h_1}, & \hat{y}_2 &= \frac{y_2}{h_2}, \\
\hat{\theta} &= \frac{T-T_{w2}}{T_{w1}-T_{w2}}, & \hat{\lambda} &= \frac{\mu_2}{\mu_1}, & \hat{\lambda}_\theta &= \frac{\psi_2}{\psi_1}, & h &= \frac{h_2}{h_1}
\end{align*}
\]

(6)

Using Eq. (6) into Eqs. (1) ~ (4), the non-dimensional governing equations become

Region 1

\[
\frac{d^2\hat{u}_1}{dy^2} + \frac{Gr}{Re} \sin(\hat{\theta}) \hat{\theta} = P
\]

(7)

\[
\frac{d^2\hat{\theta}}{dy^2} + Ec Pr \left( \frac{du_1}{dy} \right)^2 = 0
\]

(8)

Region 2

\[
\frac{d^2\hat{u}_2}{dy^2} + \frac{Gr}{Re} \frac{nb h^2}{\lambda} \sin(\hat{\theta}) \hat{\theta}_2 - \frac{M^2 h^2}{\lambda} \hat{u}_2 = \frac{h^2}{\lambda} P
\]

(9)

\[
\frac{d^2\hat{\theta}_2}{dy^2} + Ec Pr \frac{\dot{\lambda}}{\lambda_\theta} \left( \frac{du_2}{dy} \right)^2 + Ec Pr \frac{h^2}{\lambda_\theta} M^2 \hat{u}_2^2 = 0
\]

(10)

where

\[
P = \frac{h_1^2}{\mu_1 U} \frac{dp}{dx}, \quad M = B_0 h \sqrt{\frac{\sigma}{\mu_1}}, \quad Gr = \frac{\rho^2 b^2 (T_{w1}-T_{w2})}{\nu^2}, \quad Re = \frac{U h}{\nu}, \quad Pr = \frac{\mu_1 C_p}{\nu_1} \quad \text{and} \quad Ec = \frac{U^2}{C_p(T_{w1}-T_{w2})}
\]

are the dimensionless pressure gradient, the Hartmann number, the Grashof number, the Reynolds number, the Prandtl number and the Eckert number, respectively.

Accordingly the boundary and interface conditions are

\[
\begin{align*}
\hat{u}_1(0) &= 1, & \hat{\theta}_1(0) &= 1, & \hat{u}_2(0) &= \hat{u}_{20}, & \hat{\theta}_2(0) &= \hat{\theta}_{20}, \\
\frac{du_1}{dy} \bigg|_{y=0} &= \frac{\hat{\lambda}}{h} \frac{du_2}{dy} \bigg|_{y=0}, & \frac{d\hat{\theta}_1}{dy} \bigg|_{y=0} &= \frac{\hat{\lambda}_\theta}{h} \frac{d\hat{\theta}_2}{dy} \bigg|_{y=0}, \\
\hat{u}_2(-1) &= 0, & \hat{\theta}_2(-1) &= 0.
\end{align*}
\]

(11)

The asterisks for all quantities are dropped and the subscripts \( y_1 \) and \( y_2 \) in both regions are omitted for brevity.

3. METHODS OF SOLUTIONS

3.1 Analytical Solutions

The governing Eqs. (7) ~ (10) are to be solved subject to the boundary and interface conditions (11) for the velocity and temperature distributions. These equations are coupled and nonlinear because of the inclusion of the viscous and Ohmic dissipation terms in the energy equation. Hence the closed form solutions cannot be obtained. However for smaller values of the quantity \( Ec Pr (= \epsilon) \), the approximate analytical solutions can be obtained through the regular perturbation method, exploiting \( \epsilon \) to be the perturbation parameter. To this end the solutions are assumed of the form

\[
(\hat{u}_i, \hat{\theta}_i) = (u_{i0}, \theta_{i0}) + \epsilon (u_{i1}, \theta_{i1}) + ...
\]

(12)

where \( i = 1, 2 \), \( u_{i0}, \theta_{i0} \) are solutions for the case \( \epsilon = 0 \), and \( u_{i1}, \theta_{i1} \) are the perturbed quantities relating to \( u_{i0}, \theta_{i0} \), respectively. Substituting Eq. (12) into Eqs. (7) ~ (10) and equating the coefficients of like powers of \( \epsilon \) to zero, we obtain the zeroth and first order equations as follows:

Region 1

Zeroth order equations

\[
\frac{d^2\hat{u}_{10}}{dy^2} + \frac{Gr}{Re} \sin(\hat{\theta}_{10}) \hat{\theta}_{10} = P
\]

(13)

\[
\frac{d^2\hat{\theta}_{10}}{dy^2} = 0
\]

(14)

First order equations

\[
\frac{d^2\hat{u}_{11}}{dy^2} + \frac{Gr}{Re} \sin(\hat{\theta}_{11}) \hat{\theta}_{11} = 0
\]

(15)

\[
\frac{d^2\hat{\theta}_{11}}{dy^2} + \left( \frac{du_{10}}{dy} \right)^2 = 0
\]

(16)

Region 2

Zeroth order equations

\[
\frac{d^2\hat{u}_{20}}{dy^2} + \frac{Gr}{Re} \frac{nb h^2}{\lambda} \sin(\hat{\theta}_{20}) \hat{\theta}_{20} - \frac{M^2 h^2}{\lambda} \hat{u}_{20} = \frac{h^2}{\lambda} P
\]

(17)

\[
\frac{d^2\hat{\theta}_{20}}{dy^2} = 0
\]

(18)

First order equations

\[
\frac{d^2\hat{u}_{21}}{dy^2} + \frac{Gr}{Re} \frac{nb h^2}{\lambda} \sin(\hat{\theta}_{21}) \hat{\theta}_{21} - \frac{M^2 h^2}{\lambda} \hat{u}_{21} = 0
\]

(19)

\[
\frac{d^2\hat{\theta}_{21}}{dy^2} + \frac{\hat{\lambda}}{\lambda_\theta} \left( \frac{du_{20}}{dy} \right)^2 + \frac{h^2}{\lambda_\theta} M^2 \hat{u}_{20}^2 = 0
\]

(20)

The corresponding boundary and interface conditions (11) reduce to

Zeroth order conditions

\[
\begin{align*}
\hat{u}_{10}(1) &= 1, & \hat{\theta}_{10}(1) &= 1, & \hat{u}_{20}(0) &= \hat{u}_{20}, & \hat{\theta}_{20}(0) &= \hat{\theta}_{20}, \\
\frac{du_{10}}{dy} \bigg|_{y=0} &= \frac{\hat{\lambda}}{h} \frac{du_{20}}{dy} \bigg|_{y=0}, & \frac{d\hat{\theta}_{10}}{dy} \bigg|_{y=0} &= \frac{\hat{\lambda}_\theta}{h} \frac{d\hat{\theta}_{20}}{dy} \bigg|_{y=0}, \\
\hat{u}_{20}(-1) &= 0, & \hat{\theta}_{20}(-1) &= 0.
\end{align*}
\]

(21)
First order conditions

\[ u_{1i}(l) = 0 \quad \theta_{1i}(l) = 0 \]
\[ u_{11}(0) = u_{21}(0) \quad \theta_{11}(0) = \theta_{21}(0) \]
\[ du_{11}(0) = \frac{\lambda}{h} du_{21}(0) \quad d\theta_{11}(0) = \frac{\lambda}{h} d\theta_{21}(0) \]
\[ u_{21}(-1) = 0 \quad \theta_{21}(-1) = 0 \tag{22} \]

Solutions of the zeroth order Eqs. (13), (14), (17) and (18) using boundary and interface conditions (21) are

\[ \theta_{10} = c_1 y + c_2 \tag{23} \]
\[ \theta_{20} = c_3 y + c_4 \tag{24} \]
\[ u_{10} = f_1 y^3 + f_2 y^2 + f_3 y + f_4 \tag{25} \]
\[ u_{20} = d_1 \cosh(A_1 y) + d_2 \sinh(A_1 y) + l_3 y + l_4 \tag{26} \]

Solutions of the first order Eqs. (15), (16), (19) and (20) using boundary and interface conditions (22) are

\[ \theta_{11} = f_{11} y^6 + f_{21} y^5 + f_{31} y^4 + f_{41} y^3 + f_{51} y^2 + c_y + c_6 \tag{27} \]
\[ \theta_{21} = f_{11} \cosh(2A_1 y) + f_{10} \sinh(2A_1 y) + f_{17} \cosh(A_1 y) + f_{18} \sinh(A_1 y) + f_{20} \sinh(A_1 y) \tag{28} \]
\[ + f_{21} y^6 + f_{22} y^5 + f_{23} y^4 + c_y + c_6 \]
\[ u_{11} = l_1 y^7 + l_2 y^6 + l_3 y^5 + l_4 y^4 + l_5 y^3 \tag{29} \]
\[ + l_1 y^2 + d_2 y + d_6 \]
\[ u_{21} = d_1 \cosh(A_1 y) + d_2 \sinh(A_1 y) + l_{12} \cosh(2A_1 y) \tag{30} \]
\[ + l_{13} \sinh(2A_1 y) + l_{14} y^2 \cosh(A_1 y) \]
\[ + l_{15} y^2 \sinh(A_1 y) + l_{16} \cosh(A_1 y) + l_{17} y \sinh(A_1 y) \]
\[ + l_{19} y^3 + l_{10} y^2 + l_{20} y + l_{22} \]

where the values of coefficients in above equations are omitted for the sake of space. It is important to introduce the heat transfer rate at the lower and upper plates, respectively, defined as

\[ Nu_y = \frac{d\theta_y}{dy} \bigg|_{y=-1} \tag{31} \]
\[ Nu_t = \frac{d\theta_t}{dy} \bigg|_{y=-1} \tag{32} \]

where positive values of \( Nu_y \) indicate that heat transfers from the wall to the fluid and positive values of \( Nu_t \) denote the reverse.

3.2 Numerical Solutions

The analytical solutions obtained in Subsection 3.1 are valid only for small values of \( \varepsilon \). However the values of \( \varepsilon \) may not be small enough in practical applications and the validity of the perturbation method is important, it is natural to resort to numerical solutions.

The numerical method employed involves solving the governing Eqs. (7) – (10) together with boundary and interface conditions (11), using the finite difference method. Replacing the derivatives with corresponding central difference approximations of second order accuracy, we obtain finite difference equations for each layer, which is divided into 100 uniform grids. Together with boundary and interface conditions, each dependent variable is solved after a suitable number of iterations. The convergent solutions are supposed to be found when the maximum differences between two successive iterations of velocity and temperature components are less than a tolerance, \( 10^{-14} \). To validate the numerical scheme, the computed solutions are compared with analytical solutions, which are graphed in Figs. 2, 3 and tabularized in Table 1 for velocity and temperature. The numerical and analytical results are in excellent agreement for small values of \( \varepsilon \), however, the deviations between them become large for large values of \( \varepsilon \). The relative values \( (\Phi_{num} - \Phi_{ana}) / \Phi_{ana} \) at the interface \( y = 0 \) for velocity and temperature, respectively, increase from 0.00004, 0.00029, 0.032 to 0.2 and from 0, 0.0017, 0.12 to 0.46 for \( \varepsilon = 0, 0.01, 0.1 \) and 0.2. The differences between numerical and analytical solutions for large \( \varepsilon \) are expected since the regular perturbation method provides reasonably accurate results only for small perturbation parameter. To lessen the deviation between the two solutions, more number of regular perturbation terms is required. Therefore the figures in the present study are drawn using results of numerical results. Physically large value of \( \varepsilon \) reflects to certain amount of heating generated due to dissipations, which implies a greater temperature rise in the two-fluid system. The velocity profiles are correspondingly enhanced as a result of the increase of the upward thermal buoyancy force, as the second terms observed in the left-hand sides of Eqs. (7) and (9).

4. RESULTS AND DISCUSSION

Two-fluid magnetohydrodynamic Poiseuille-Couette flow and heat transfer in an inclined channel is investigated analytically by regular perturbation method and numerically by finite difference technique. Eqs. (7) – (10) subject to Eq. (11) for the velocity and temperature distributions are solved numerically and the results are depicted graphically in Figs. 4 to 11. Since the problem involves many parameters, we fix some of them, namely \( P = -5.0, Pr = 0.7, Re = 1.0, b = 1, n = 1.0 \), for all the computations and analyze the effect of other important non-dimensional parameters on the flow and heat transfer characteristics. It is seen from Figs. 2, 3 and Table 1 that analytical and numerical solutions are in a good agreement for small values of the product of the Prandtl number and Eckert number (\( = \varepsilon \)) and the difference becomes large as \( \varepsilon \) increases. It is also observed that as \( \varepsilon \) increases both the velocity and temperature increase. The increase of temperature resulting from the increasing dissipation effect due to large \( \varepsilon \) is evident, and as a consequence the velocity increases for the increasing buoyancy force in the momentum equation.
The effect of magnetic field on the velocity is shown in Fig. 4. The effect of increasing magnetic field is to decrease the velocity, reflecting the overall retarding effect of the Lorentz force. The Hartmann number represents the ratio of the Lorentz force to the viscous force, implying that the larger the Hartmann number, the stronger the retarding effect on velocity field. The effect of $M$ on temperature is not significant in the present study, as seen from Table 2. Figure 5 depicts the
effect of Grashof number on the velocity. An increase in the value of Grashof number $Gr$ increases the velocity. Physically an increase in the value of Grashof number implies an increase of buoyancy force which supports the motion. Since the Grashof number acts as a driving mechanism of the buoyancy force in the momentum equation, the velocity and/or velocity gradient increases and therefore the effect of dissipations increases, resulting in the enhancement of temperature fields in both the regions as noted in Table 2. The effect of inclination angle $\phi$ can be observed in Fig. 6. It is found that the increase in the value of $\phi$ increases the velocity. This is due to the fact that magnitude of the driving force increases with the inclination angle. From Table 2, it is evident that the effect $\phi$ on temperature is not sensitive.

The effect of the ratio of viscosities $\lambda (\mu_2 / \mu_1)$ on the velocity field is depicted in Fig. 7. Since $\lambda$ is the ratio of viscosity of the conducting fluid to that of non-conducting fluid, increasing the value of $\lambda$ indicates that viscosity of the conducting fluid is larger than that of clear fluid. Thus it is obvious that the velocity in region-2 is reduced for the case of increasing $\lambda$. The velocity of fluid in the upper region, though experiencing the driving force of the upper plate, is also lowered down due to the dragging condition at the interface for $\lambda > 1$. The effect of $\lambda$ is to enhance the temperature field as noticed in Table 2. Even though the increasing in $\lambda$ reduces the velocity and/or velocity gradient, however, the total effects of viscous and Ohmic dissipations on temperature fields are increased through the enlargement of $\lambda$. It is also noted that the influence of $\lambda$ is not very effective on temperature field. The effect of height ratio $h(h_2/h_1)$ is to promote both the velocity and temperature as shown in the respective Figs. 8 and 9. It is seen that the larger the height of the conducting fluid compared to the non-conducting fluid, the larger the flow and temperature fields. The effect of the thermal conductivity ratio $\lambda_T (\varphi_2 / \varphi_1)$ on the velocity and temperature fields is displayed in Figs. 10 and 11, respectively. As the thermal conductivity of the conducting fluid compared to the non-conducting fluid increases, the velocity and temperature distributions reduce. It can be observed from Eq. (10) that increasing $\lambda_T$ reduces the viscous and Ohmic dissipations and hence the temperature is suppressed, which also results in a suppression of velocity field. It is also found that effect of $\lambda_T$ is highly significant on temperature profile compared to that on velocity.

The effects of various physical parameters on the Nusselt numbers at the upper and lower plates of the channel are given in Table 3. The parameters such as Prandtl number, Reynolds number, pressure parameter, inclination angle, density ratio and conductivity ratio are fixed as 0.7, 1, $-5$, 30°, 1 and 1, respectively. It is observed that the Nusselt numbers at the lower plate $Nu_-$ are increased for increasing values of Eckert number $Ec$, Grashof number $Gr$ and height ratio $h$ but are decreased for increasing values of Hartmann number $M$, viscosity ratio $\lambda$ and thermal conductivity ratio $\lambda_T$. On the other hand, increasing the values of $Ec$, $Gr$ and $h$
reduces the Nusselt number at the upper plate $Nu_u$, and it is increased for increasing $M$, $\lambda$ and $\lambda_T$. It is also noted that Nusselt number at the lower plate is always positive, indicating that heat always transfers from the fluid to the plate due to the less temperature specified at the lower plate. And negative values of $Nu_u$ implies that the temperature of the fluid is higher than that of the upper plate due to the heating effect of dissipations.

5. CONCLUSIONS

The flow and heat transfer aspects of the combination of conducting and non-conducting immiscible fluids in an inclined channel, keeping the upper plate moved with a constant velocity and the flow driven by constant a pressure gradient, are analysed. The governing equations are solved analytically valid for small $Pr Ec$ and numerically valid for all values of $Pr Ec$. It is found that the velocity is enhanced by increasing the values of $\epsilon$, Grashof number, angle of inclination and height ratio, whereas it is suppressed by increasing the values of Hartmann number, viscosity and conductivity ratios. The Nusselt number at the upper plate is increased by increasing the values of $M$, $\lambda$ and $\lambda_T$, and it is decreased by increasing $Ec$, $Gr$ and $h$. The lower plate Nusselt number is reversely affected by the relevant parameters.

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